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Damage Detection in Beam Structures Using Subspace Rotation Algorithm with Strain Data

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Motivated by a smart structures philosophy of self-health monitoring in structural systems, a damage detection system for beam structures using subspace rotation techniques is presented. In doing so, the subspace rotation is extended from displacement-based data to strain-based data. Methods for accommodating the additional degrees of freedom of beam elements are presented. Using simulated data, the ability to locate and quantify single and multiple damage events is investigated. It has been found that the higher-order vibration modes are required to locate damage events in beams because the frequency response of beams is much less sensitive to a damage event than that of truss structures. It has also been found that condensation methods cannot be used to remove rotational degrees of freedom in the beam because of the coupled nature of the rotational degrees of freedom to the translational degrees of freedom.

I. Introduction

THE detection of damage as a part of self-health monitoring in structural systems is important in increasing the safety and reliability of structures and structural components, as well as providing the potential for reducing life cycle costs and improving repair turnaround times. If damage can be located in a structure and its evolution can be observed, then components of the structure can be repaired or replaced before some critical point is reached and a dangerous and/or costly failure occurs. The damage detection system can potentially reduce maintenance costs by enabling maintenance for cause instead of maintenance by schedule. Furthermore, the damage detection system can reduce injury and loss of life and the associated cost of liability, because it would provide an on-line early warning capability.

For any structural system there are many different damage modes. For composite systems some of these damage modes include reduction in area, matrix cracking, fiber breakage, debonding, and delamination.[1] For aluminum systems, some damage modes include reduction in area, corrosion, fatigue cracking, and brittle and/or ductile fracture. These damage modes can be caused by a number of potential load scenarios: low- and high-velocity impact, vibrations, fatigue, overload, thermal/environmental effects, etc. No matter what the damage mode or load scenario, when a structure becomes damaged in some way, there are changes in geometry and/or material properties. For this paper, damage is therefore defined as some change in the parameters of a structural model.

One class of damage detection methods in which damage is seen as a change in the parameters of a structural model is based on modal information. The modal-based damage detection methods use a finite element model of the system modal response coupled with experimental modal data to determine damage location and extent. The term damage location means the spatial location of the damage event on the structure relative to a predetermined coordinate reference frame. The term damage extent denotes the size of the metric used in describing the severity of damage. The precise definition of the damage metric depends on the damage mode and damage detection algorithm. Zimmerman and Weaver Smith[2] provide an excellent review of the modal-based damage detection systems, so rather than repeat their efforts this paper will review the literature most pertinent to the present application.

In 1992 Kashangaki[3] described damage detection using a sensitivity-based eigenstructure assignment method in which he separated the damage location and damage extent problems. Kashangaki showed that this algorithm could detect the occurrence, location, and magnitude of damage in a truss structure. The algorithm could not, however, detect every damage event that was imposed, particularly multiple damage events. The author pointed out that some modifications to the method were necessary before real-time implementation could be possible. Also, in 1992 Zimmerman and Kaouk[4] presented a very similar method that they termed "subspace rotation algorithm." This method also separated the damage location and damage extent problems. Both of these subproblems required extremely simple mathematical manipulations; therefore, this damage detection algorithm was computationally inexpensive. Zimmerman and Kaouk demonstrated that this algorithm could detect the spatial location and extent of a damage event or damage events in a truss structure using perfect eigenvalue and eigenvector data.

Another method based on a structural model and changes in the model parameters due to damage was presented by Lindner et al.[5] in 1993. They recognized that damage to a truss element may cause a significant change in one or more of the entries of the finite element model's stiffness matrix. They proposed an algorithm that 1) parameterized the damage in such a way that the reduction in stiffness in an individual element was reflected in the global stiffness matrix and 2) calculated the percent reduction of the stiffness of a strut from an output pole placement formulation.[6] They did not use separate algorithms to locate damage and determine its extent, but their technique of looking at each element individually made their method very similar to the methods of Kashangaki[3] and Zimmerman and Kaouk.[4]

These approaches to damage detection demonstrated the ability to locate damage and determine its extent in spring-mass systems and in truss structures using displacement data.[3–5] This paper explores the feasibility of extending the subspace rotation approach to damage detection to beam structures using strain data. We have concentrated on beam structures as a first step in exploring the added complexity of applying subspace rotation algorithms to nontruss structures. Investigating the use of strain data was originally motivated by added mass and cost concerns (strain gauges are generally less costly and weigh less than accelerometers). However, as will be shown in Sec. IV, using strain information also provides a means of overcoming the coupled nature of translational and rotational degrees of freedom in beam elements.

II. Subspace Rotation Damage Detection Algorithm

The subspace rotation algorithm[4] attempts to provide a unique solution by viewing damage location and damage extent as two different problems requiring two separate algorithmic solutions. By

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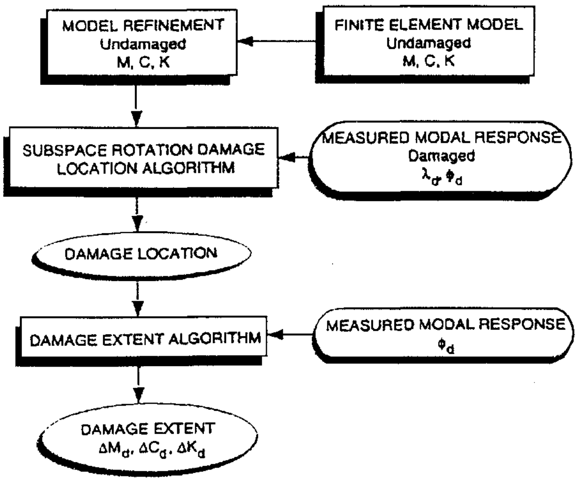

Fig. 1 Subspace rotation algorithm.

decoupling damage location and damage extent, not only can a unique damage location be found, but each subproblem is computationally more attractive because each subproblem requires only matrix-scalar and matrix-vector multiplications, with one matrix inversion for the damage extent algorithm. For the sake of clarity and continuity, this section reviews the subspace rotation damage detection algorithms originally developed by Zimmerman and Kaouk.[4] Subsequent sections will describe the use of these algorithms with beam elements and strain data.

With the subspace rotation algorithm, structural damage is located by identifying discrepancies between the original finite element model modal properties and the postdamaged modal properties. A flow chart of the subspace rotation algorithm is shown in Fig. 1. Essentially, a refined finite element model of a healthy structure is used in conjunction with the measured modal parameters of a damaged structure. With this information, the subspace rotation algorithm provides the damage location. Once the damage is located, that information is used with the refined finite element model and the measured modal parameters to determine the extent of the damage.

A. Damage Location

The damage location algorithm, as developed by Zimmerman and Kaouk,[4] starts with an n-degrees-of-freedom finite element model of the undamaged structure:

$$Mx_{tt} + Cx_t + Kx = 0 \tag{1}$$

where M is the $n \times n$ ideal mass matrix, C is the $n \times n$ ideal damping matrix, K is the $n \times n$ ideal stiffness matrix, x is the $n \times 1$ displacement vector, and the subscript t denotes differentiation with respect to time. By assuming that the displacement vector is a harmonic solution of the form $x = \phi e^{-j\omega t}$, an associated eigenproblem is obtained:

$$\left(-\omega_{hi}^2 M - j\omega_{hi} C + K\right)\phi_{hi} = 0 \tag{2}$$

where ω_i is the ith natural frequency and ϕ_i is the corresponding mode shape($i = 1, \ldots, n$). The subscript h indicates a healthy predamaged structure in which the natural frequencies and mode shapes satisfy the ideal eigenproblem.

Zimmerman and Kaouk[4] then assumed that damage to the structural system is manifested by changes of the property matrices from (M, C, K) to $(M - \Delta M_d, C - \Delta C_d, K - \Delta K_d)$ with associated changes of the natural frequencies and mode shapes from (ω_{hi}, ϕ_{hi}) to (ω_{di}, ϕ_{di}), where the subscript d denotes a damaged structure. The damaged structure then satisfies the following equation:

$$\left[-\omega_{di}^2(M - \Delta M_d) - j\omega_{di}(C - \Delta C_d) + (K - \Delta K_d)\right]\phi_{di} = 0 \tag{3}$$

Because the perturbation matrices $(\Delta M_d, \Delta C_d, \Delta K_d)$ are assumed to be exact, Eq. (3) holds for any set of measured natural frequencies and mode shapes. This assumption is important because it may be impractical or impossible to measure every mode with complex structural components. The algorithm must produce reliable results with only p measured modes, where $p \leq n$. From this point forward in the discussion i denotes the measured modes ($i = 1, \ldots, p$). Equation (3) can be rearranged leaving the original matrices on one side and moving the perturbation matrices to the other side:

$$\left(-\omega_{di}^2 M - j\omega_{di} C + K\right)\phi_{di} = \left(-\omega_{di}^2 \Delta M_d - j\omega_{di} \Delta C_d + \Delta K_d\right)\phi_{di} \tag{4}$$

which can be simplified to

$$d_i^1 = d_i^2 \tag{5}$$

where

$$d_i^1 = \left(-\omega_{di}^2 M - j\omega_{di} C + K\right)\phi_{di} \tag{6}$$

and

$$d_i^2 = \left(-\omega_{di}^2 \Delta M_d - j\omega_{di} \Delta C_d + \Delta K_d\right)\phi_{di} \tag{7}$$

In the case of no damage, the perturbation matrices are all zero so that by Eq. (7), $d_i^2 = 0$, which in turn means that $d_i^1 = 0$ because of Eq. (5). If, on the other hand, some damage has occurred, values in certain rows of the associated perturbation matrix will be nonzero, which in turn will cause zero/nonzero values in d_i^2 and d_i^1 corresponding to the rows of the perturbation matrices. This provides an indication of the damage location because the row of the damage vector affected by damage corresponds to the degree of freedom of the finite element model that is affected by damage. Notice that even though the preceding discussion was based on the behavior of the perturbation matrices, it is not necessary to actually know these perturbation matrices to implement the concept. Instead, d_i^2 can be found with knowledge of the original property matrices and the damaged mode shapes and natural frequencies because, according to Eq. (5), $d_i^2 = d_i^1$.

The critical results about this damage location algorithm to be noted are the following: 1) damage can be located by identifying the location of nonzero values in the damage vector of Eq. (6); 2) the algorithm is computationally efficient because only simple matrix multiplications are required to locate damage in structural systems; 3) the finite element model is required only to determine the original undamaged property matrices (M, C, K) used in Eq. (6); and 4) not every mode from the finite element model needs to be measured; only a few arbitrarily chosen postdamaged natural frequencies and corresponding mode shapes (ω_{di}, ϕ_{di}) must be measured. These simple attributes form the cornerstone of the subspace rotation algorithm.

B. Damage Extent

In looking at the finite element model of the structure, the extent of damage, as developed by Zimmerman and Kaouk,[4] can be determined by the perturbation matrices $(\Delta M_d, \Delta C_d, \Delta K_d)$ such that Eq. (3) is satisfied. For clarity of presentation, it is assumed that damping may be neglected and that damage initiation does not alter the mass of the structure. Making this assumption simplifies the problem by reducing the number of required matrix permutations by one third. As a result, the extent of damage is embodied in the change in the stiffness matrix (ΔK_d). With these assumptions, Eq. (3) can then be reduced and rewritten:

$$\left(-\omega_{di}^2 M + K\right)\phi_{di} = \Delta K_d\, \phi_{di} \tag{8}$$

Zimmerman and Kaouk[4] assumed that the change in the stiffness matrix (ΔK_d) can be viewed as a pseudo-output feedback controller[7]:

$$\Delta K_d = B_o G \tag{9}$$

where B_o is the $n \times p$ pseudocontrol influence matrix and $G = FC$, where F is the $p \times p$ pseudofeedback gain matrix and C is the $p \times n$ pseudo-output influence matrix. The idea behind this concept is that one may cast the damaged system of equations into a controller feedback formulation, with the controller weights being related to nondimensional internal damage state variables.[7]

Zimmerman and Kaouk[4] chose the pseudocontrol influence matrix to be equal to the damage location vectors,

$$B_o = [d_1 \quad d_2 \quad \cdots \quad d_p] \tag{10}$$

because the pseudocontrol forces will directly influence only those degrees of freedom that are affected by damage as indicated by the d_i [note that the superscript 1 used in Eq. (6) has been dropped for simplicity]. Also, the updated stiffness matrix is required to be symmetric. This is accomplished by requiring that

$$G = HB_o^T \tag{11}$$

where H is a $p \times p$ arbitrary matrix. With appropriate equation substitutions and rearrangements, Zimmerman and Kaouk[4] found this arbitrary matrix to be

$$H = \left(B_o^T \Phi_d\right)^{-1} \tag{12}$$

With further substitutions, they found the following result for the change in the stiffness matrix ΔK_d:

$$\Delta K_d = d\left[d^T \Phi_d\right]^{-1} d^T \tag{13}$$

where d is the matrix of damage vectors $[d_1 \ d_2 \ \cdots \ d_p]$ and Φ_d is the matrix of mode shapes $[\phi_{d1} \ \phi_{d2} \ \cdots \ \phi_{dp}]$. Thus, the extent of the damage can be determined.

III. Subspace Rotation Algorithm on a Cantilevered Beam

Now the extension of the subspace rotation algorithm to beam structures from truss structures will be demonstrated. A cantilevered beam has been chosen as a simple continuous system representative of some realistic structure for which a damage detection system would be useful. The beam is arbitrarily assumed to be made of aluminum (6061-T6) with dimensions and material properties as shown in Fig. 2.

A. Structural Model Development for Beam Elements

To use the subspace rotation algorithm, the structural model of this cantilevered beam must first be determined. The development for an Euler–Bernoulli beam in bending given in finite element and vibrations analysis literature[8] is followed.

A beam element is shown in Fig. 3. At each end there are two local degrees of freedom, one for vertical translation v and one for rotation θ making a total of four degrees of freedom, $v_1, \theta_1, v_2, \theta_2$, per element. A truss element also contains four degrees of freedom, a vertical and a horizontal translation at each end, but the vertical and horizontal translations are not coupled to one another as the vertical translations are to the rotations of the beam element. The structural model representative of a beam element with no damping must, therefore, contain a 4×4 mass matrix and a 4×4 stiffness matrix that couple the vertical translations to the rotations. It is given

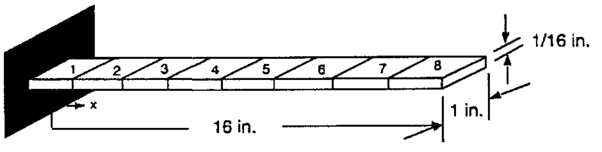

Fig. 2 Analytical test article: $E = 10 \times 10^6$ psi; $I = 2.0345 \times 10^{-5}$ in.^{4}; $\gamma = 0.098$ lb/in.^{3}; and $\rho = 2.54 \times 10^{-4}$ lb mass/in.^{3}

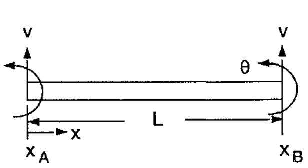

Fig. 3 Beam element.

in Thompson[8] that for a uniform beam element the elemental mass and stiffness matrices are

$$m^e = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{14}$$

and

$$k^e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{15}$$

These elemental matrices can be assembled into global matrices representing the entire beam, where coefficients of the elemental matrices of one beam element are added to the coefficients of the elemental matrices of another beam element that shares a common node. Thus, the structural model of a cantilevered beam necessary for the subspace rotation algorithm can be determined.

B. Condensation

With the beam shown in Fig. 2, two of the degrees of freedom of the structural model are translations and two are rotations. It is often difficult to measure rotational responses or excitations. As a result, the rotational degrees of freedom are generally not measured unless it is absolutely necessary.[9] One way of overcoming this difficulty is to avoid altogether making measurements of the rotational degrees of freedom. Condensation techniques can be used with finite element models to eliminate the rotational degrees of freedom, thereby making their measurement unnecessary. This approach is often used in the context of model-based damage detection to reduce the complexity of the structural model when it is too costly to measure or model all degrees of freedom. In the case of bar elements used in trusses, only translational degrees of freedom are used in the models, so condensation simply reduces the number of translational degrees of freedom that must be measured. With beam elements the issue is not necessarily to reduce the number of degrees of freedom; rather it is to eliminate a type of degree of freedom, namely rotations. This is an important distinction because rotational and translation degrees of freedom are coupled in elemental stiffness matrices of beams. As will be shown in the discussion that follows, this coupling makes the standard practice of reducing the model not possible with beam elements.

One of the simplest and most common methods used for eliminating the rotational degrees of freedom is a static condensation method known as Guyan reduction.[10] This method partitions the structural model into master m degrees of freedom, which are to be retained, and slave s degrees of freedom, which are to be condensed out as follows:

$$\left\{\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix}\right\}\begin{Bmatrix} \phi_m \\ \phi_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{16}$$

Following the development in Reddy,[10] the reduced-system mass and stiffness matrices (M^* and K^*) are

$$M^* = T^T MT \tag{17}$$

and

$$K^* = T^T KT \tag{18}$$

where

$$T = \begin{Bmatrix} I \\ -K_{ss}^{-1}K_{sm} \end{Bmatrix} \tag{19}$$

Ideally, the subspace rotation algorithm would work as well with the reduced system as it does with the full system, as has been demonstrated with truss structures.[2] If so, the need to measure rotations would be eliminated. While seemingly attractive, this approach fails to work because of the coupled nature of the translational and rotational degrees of freedom in the mass and stiffness matrices of the beam elements.

The basic assumption behind the subspace rotation algorithm is that the governing equation of motion, Eq. (3), can be rearranged and rewritten so that the original matrices remain on one side of the equal sign and the change in the matrices can be moved to the other side of the equal sign as in Eq. (4). This cannot be done with the reduced system, as can be easily shown by performing condensation and then formulating the governing equation of motion of the reduced system:

$$\left[(K - \Delta K)^* - \omega_d^2 M^*\right]\Phi_d = 0 \tag{20}$$

The change in stiffness ΔK^* should now be moved to the right-hand side. However, moving ΔK^* requires first expanding the $(K - \Delta K)^*$ term to

$$(K - \Delta K)^* = (K_{mm} - \Delta K_{mm}) - (K_{ms} - \Delta K_{ms}) \times (K_{ss} - \Delta K_{ss})^{-1}(K_{sm} - \Delta K_{sm}) \tag{21}$$

Notice that the second term on the right-hand side of Eq. (21) contains terms of the form $K \times \Delta K$ and, therefore, ΔK-type terms cannot be isolated from K-type terms and moved to the right-hand side of Eq. (20). More simply put,

$$(K - \Delta K)^* \neq K^* - \Delta K^* \tag{22}$$

The subspace rotation algorithm requires that the stiffness and change of stiffness terms be separated, so the algorithm cannot be used with the reduced model because of these coupled terms. One possible method of dealing with the rotational degrees of freedom that does not require reduction techniques is described in Sec. IV.A.

IV. Subspace Rotation Algorithm with Strain Data

The modal information used in the subspace rotation algorithm is generally not directly measured; it is extracted from some measured data. This paper uses a collection of strain frequency response functions (measured at the center of each beam element) from which both the rotational and translational degrees of freedom are determined. As will be seen in the development that follows, this approach enables the rotational degrees of freedom to be determined indirectly, thereby enabling the use of subspace rotation with beam elements. One might reasonably ask at this point why the strain data are not used directly in the subspace rotation algorithm. It would appear that simple differentiation would lead to a strain equivalent of Eqs. (4) and (13). However, differentiation of this type would replace translations and rotations with strains and curvatures, which would not alleviate the issue of coupled degrees of freedom.

A. Extracting Modal Information from Strain Data

One of the most commonly used methods of acquiring modal information is to extract the natural frequencies and modes shapes from displacement frequency response functions using the circle-fit method.[9] This approach is used in this paper as well because it is well documented and understood. If strain sensors are used, then the data will be in the form of strain frequency response functions. This section discusses a method of determining displacement frequency responses from strain frequency responses, so the modal information can still be extracted from displacement frequency response functions using the circle-fit method.

The basic concept of obtaining displacement data from strain data lies in assuming this is a linear elastic problem. For a linear elastic system, a load is defined as the product of the system stiffness and its displacement:

$$Q = Ku \tag{23}$$

If the load vector Q and the stiffness matrix K are known, then the displacement vector u can be found by

$$u = K^{-1}Q \tag{24}$$

Generally, the load vector is some vector of applied forces, but for the purposes of this paper the load vector is a vector of measured strain values.

To relate strains to displacements, the finite element method is used. This relation is accomplished by applying the principal of minimum potential energy to formulate elemental stiffness matrices and elemental load vectors.[11] Following the method outlined in Cook,[11] the following is determined for the 4×4 elemental stiffness matrix k^e and the 4×1 elemental load vector Q^e that contains the measured strain information:

$$k^e = \int_V \left(\psi_{xx}^e\right)^T E y^2 \left(\psi_{xx}^e\right) dV \tag{25}$$

and

$$Q^e = -\int_V \left(\psi_{xx}^e\right)^T E y \epsilon_o^e \, dV \tag{26}$$

where E is the modulus of elasticity, y is the distance from the beam's neutral axis to the point of interest, ψ_{xx}^e is a 1×4 vector of second derivatives of the shapes functions with respect to x, and ϵ_o^e is the measured strain value.

The solution to k^e is

$$k^e = EI \int_0^L \left(\psi_{xx}^e\right)^T \left(\psi_{xx}^e\right) dx \tag{27}$$

where I is the moment of inertia and L is the length of the element. The solution to this equation is the 4×4 elemental stiffness matrix for an Euler–Bernoulli beam given in Eq. (15).

The solution to Q^e requires an additional substitution before the integration. The initial strain term is defined as

$$\epsilon_o = \sigma/E = My/EI \tag{28}$$

where M is the bending moment. With $y = h/2$, where h is the thickness of the beam element, the strain on the surface is defined as

$$\epsilon_{os} = Mh/2EI \tag{29}$$

Rearranging this equation to find the bending moment and substituting into Eq. (28) yields

$$\epsilon_o = (2y/h)\epsilon_{os} \tag{30}$$

With this substitution,

$$Q^e = -EI\left(\frac{2}{h}\right)\epsilon_{os}^e \int_0^L \left(\psi_{xx}^e\right)^T dx \tag{31}$$

which is

$$Q^e = \frac{2EI}{h}\epsilon_{os}^e \begin{Bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{Bmatrix} \tag{32}$$

This elemental load vector can be assembled into a global load vector. For a cantilevered beam with eight elements and eight corresponding strain values $(\epsilon_{os}^1, \epsilon_{os}^2, \ldots, \epsilon_{os}^8)$, the global load vector is

$$Q = \frac{2EI}{h} \begin{Bmatrix} 0 \\ \epsilon_{os}^2 - \epsilon_{os}^1 \\ 0 \\ \epsilon_{os}^3 - \epsilon_{os}^2 \\ 0 \\ \epsilon_{os}^4 - \epsilon_{os}^3 \\ 0 \\ \epsilon_{os}^5 - \epsilon_{os}^4 \\ 0 \\ \epsilon_{os}^6 - \epsilon_{os}^5 \\ 0 \\ \epsilon_{os}^7 - \epsilon_{os}^6 \\ 0 \\ \epsilon_{os}^8 - \epsilon_{os}^7 \\ 0 \\ -\epsilon_{os}^8 \end{Bmatrix} \tag{33}$$

Again, this load vector holds the strain information. With the strains known and the displacements unknown, the displacement vector can be found by Eq. (24). Note that a load vector for a cantilevered beam that contains 8 strain values produces a displacement vector that contains 16 displacement values, 8 translations, and 8 rotations. This is important because the rotations are determined without having to measure them. With the displacement frequency responses known, the modal information can be extracted using the circle-fit method.[9]

The circle-fit method begins with setting up a window around a point of resonance of a frequency response function by choosing a small range of frequencies above and below the resonance frequency. The natural frequency is determined, and a circle is curve-fitted to the measured data with the window. The modal parameter and mode shape value are then determined. This process is repeated for another mode of interest. Note that to determine the full mode shape for each mode the whole process must be repeated for each degree of freedom of the beam's structural model.

B. Subspace Rotation Algorithm with Simulated Strain Data

This section presents the results of simulating strain data and using the extracted information with the subspace rotation algorithm. Plots of the damage vectors are shown and are interpreted to determine the ability of the subspace rotation algorithm to locate and quantify a given damage event using strain data.

For a given damage event to an element in the cantilevered beam, the strain frequency response functions that would be measured are numerically simulated. Next, the displacement frequency response functions are found by using Eq. (24) for each degree of freedom at every frequency for which the data were simulated. A circle fit is performed for the first six modes of each displacement frequency response function to determine the natural frequencies and mode shapes. Finally, the natural frequencies and mode shapes are used with the damage location algorithm, Eq. (6), and if the damage is located, with the damage extent algorithm, Eq. (13).

Three examples are presented. In each case, element 4 is damaged, first by a 95% stiffness reduction, then by a 30% stiffness reduction, and finally by a 5% stiffness reduction. Plots of the components of the damage vectors (d_i) are presented, and if the damage is correctly located, the extent of the damage is given. Each graph contains the first six damage vectors plotted against the node numbers. Each damage vector is normalized by its maximum absolute value and plotted between zero and one. To present all six damage vectors on one plot, the damage vectors are scaled so that the mode 1 damage vector lies between zero and one, the mode 2 damage vector lies between one and two, and so on. Recall that the damage location algorithm is formulated such that the damage vector values at the nodes of an element with no damage are zero and the values at the nodes of an element with damage are one (or some other nonzero value in the case of multiple damage events).

Figure 4 shows the first six damage vectors for a 95% stiffness reduction in element 4. The damage is located by modes 2–6. The

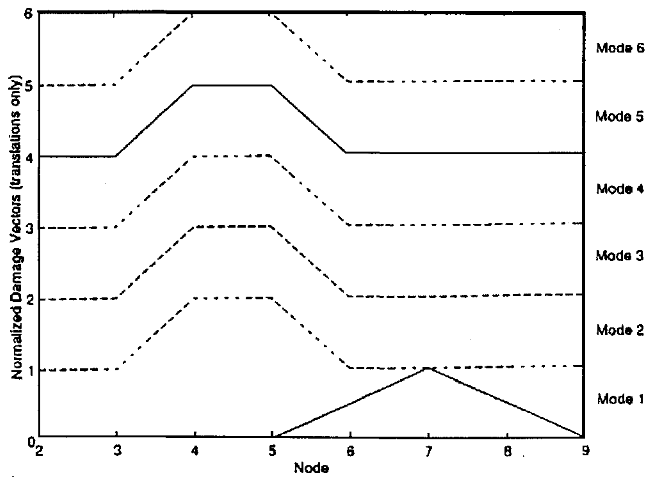

Fig. 4 Element 4, 95% damage. First six damage vectors (from the first six modes).

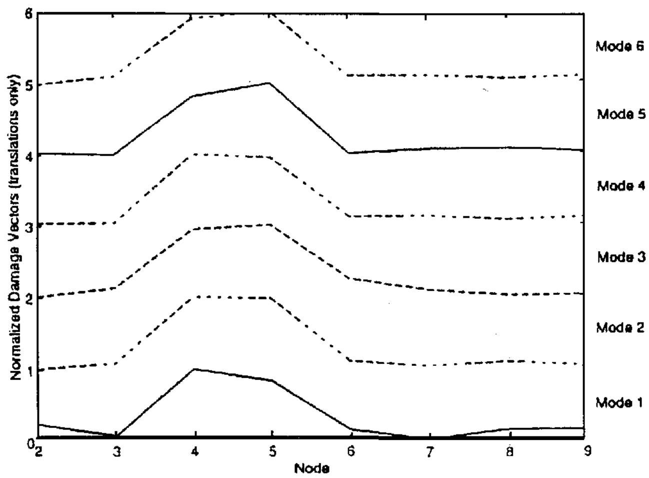

Fig. 5 Element 4, 30% damage. First six damage vectors (from the first six modes).

inability of mode 1 to detect damage in this case is attributable to the fact that the damage at element 4 has limited effect on the vibrational modes. Through a series of parametric studies, it was determined that mode 1 rarely located the damage event (30% damage at element 4 is one of the few exceptions). Using modes 2–6, the damage extent is predicted to be 95.06%. For a large percentage of damage, the damage event is located and the damage extent is accurately predicted.

Figure 5 shows the first six damage vectors for a 30% stiffness reduction in element 4. The damage is located by all six modes; however, the damage vectors do not clearly demonstrate the zero/nonzero pattern produced with error-free modal data. Furthermore, the damage extent is not correctly predicted by modes 1–6. Modes 2–4 appear to be the best locators of the damage. Using only these modes to calculate the damage extent, the predicted extent of the damage is 32.57%. Again, the damage is located, but only by those modes most affected by the damage event.

Figure 6 shows the first six damage vectors for a 5% stiffness reduction in element 4. In this case the damage is not clearly located, and therefore no prediction of the damage extent is made. For a small percentage of damage, the damage event cannot be located because of the error associated with the natural frequencies and mode shapes calculated using the circle-fit method.

Numerical error analysis was performed on the proposed modification of the subspace rotation algorithm by adding various degrees of random error to the simulated strain frequency response functions. This was done for many combinations of damage location, damage extent, mean strain error, and standard deviation of strain error. In this way the propagation of error through the numerical algorithm could be investigated. The results of these investigations

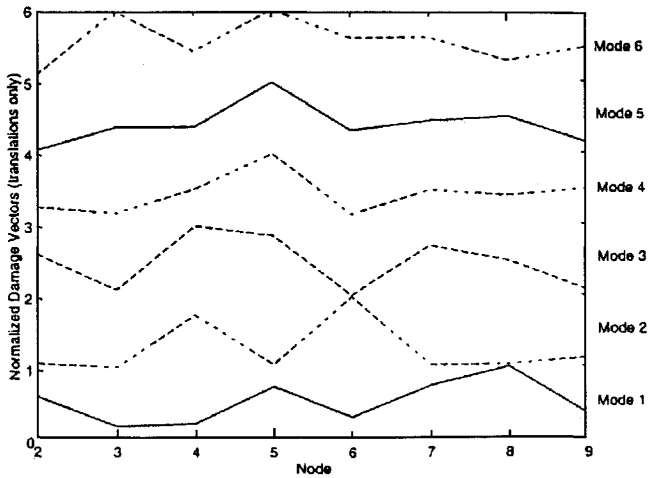

Fig. 6 Element 4, 5% damage. First six damage vectors (from the first six modes).

are documented in Ref. 12, and simply served to reinforce intuitive limits on the method to locate and quantify damage in the presence of noise. As a result, these results are not reproduced in the present paper.

V. Conclusion

This paper has presented a method of locating and quantifying damage events in beam structures using strain data. The subspace rotation algorithm for damage prediction was described and developed for beam elements. A method was also presented to extract modal information necessary for the subspace rotation algorithm from measured strain data.

There was an important finding associated with using the subspace rotation algorithm to predict damage in a cantilevered beam. Unlike for truss structures, the translational degrees of freedom are coupled to the rotational degrees of freedom in a beam such that the subspace rotation algorithm does not work when the rotational degrees of freedom are condensed out. The result of this finding is simply that rotational as well as translational degrees of freedom must be found and used with the subspace rotation algorithm.

Another important finding was that the subspace rotation algorithm has been shown to locate some damage events and predict the damage extent using a structure's modal information extracted by means of strain measurements. The relative insensitivity of the modal response of beams to certain types of damage leads to the conclusion that as many modes as possible must be used to locate the damage event to add a degree of statistical confidence (this conclusion reaffirms the findings of many previous authors). Therefore, collections of damage vectors should be used to determine whether a structure is damaged. This requirement must be balanced with the added expense of experimentally obtaining accurate higher-order-mode data. However, even using a collection of damage vectors, small amounts of damage are not located with the method described in this paper. Also, the modes that most clearly locate the damage must be used to predict the damage extent.

An advantage to using measured strain data is that the strain formulation is such that both the vertical and rotational displacements at each node of an element are determined from a single strain measurement at the center of the element; therefore, one strain value leads to four displacement values. For the cantilevered beam example, there are 16 measurable displacements, but only 8 strain measurements are required (1 at the center of each element) to calculate all the displacements; hence, only half the number of sensors are necessary to calculate displacements using the strain formulation as are necessary when simply measuring the displacements. Obtaining translations and rotations from measured strain values is also important because it eliminates the need to measure rotational degrees of freedom.

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